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Pre – Hawking Radiating Gravitational Collapse in Stationary Space-Time

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ABSTRACT: We studied the most general spherically symmetric space-time, in time – independent gravitational fields in related to the pre - Hawking radiation, gravitational collapse and stability of the model. It is seen that in stationary space-time, there is no pre - Hawking radiation, and model will continue to collapse due to the gravitational effect. The model is unstable and has no stability phase in stationary space-time.

Keywords: Einstein Field Equations, Gravitational Collapse, Hawking Radiation.

I. INTRODUCTION

General relativity has developed into an essential tool in modern Astrophysics. It is the simplest theory that is consistent with the experimental data. It is the basis of current cosmological models of a consistently expanding universe and it provides the foundation for the current understanding of gravitational collapse and black holes.

A massive star can collapse under the force of its own gravity. The final fate of such a continuous gravitational collapse will be either a black hole or a naked singularity under different conditions in general theory of relativity. Stephan Hawking [1] has theoretically predicted that black holes are weak emitters of thermal radiation generated close to the event horizon. The event horizon is the boundary where light is forever trapped by the black hole's gravitational pull. The thermal radiation emitted from the black holes is called Hawking radiation. It arises due to the quantum effects near the event horizon. The horizon separates the virtual particle pair (constantly being created from the quantum vacuum) such that one of them is sucked inside the black hole and the other one escapes, causing the black hole to lose energy. Hawking radiation reduces the mass and the energy of the black hole and is therefore also known as Black Hole Evaporation. The event horizon is supposed to mark a boundary beyond which nothing can escape a black hole's gravity. According to general relativity, even light is trapped inside the horizon and no information about what fell into the hole can ever escape. Thus, the Hawking radiation from a black hole does not carry back any information about the hole. So, in this sense, the information seems to have been destroyed. However, this contradicts quantum mechanics which holds that information inside systems can't be destroyed. The equations of quantum mechanics always preserve information. This creates the "information paradox" of black holes. But recently some researchers argue the information may never have cut off in the first place. Vachaspati *et. al.* [2] have tried to calculate what happens as a black hole is forming. They find that the gravity of the collapsing mass starts to disrupt the quantum vacuum, generating what they call pre - Hawking radiation. Losing that radiation reduces the total mass – energy of the object – so that it never gets dense enough to form an event horizon and a true black hole. Their study says that the true event horizon never forms in a gravitational collapse. Rather than forming a full – fledged event horizon from which light can never escape, the black hole can have a state of everlasting collapse that could last a very, very long time for large stars. Thus no event horizon means that nothing is cut off from the rest of the universe. Hence there is no information paradox.

Time – independent gravitational fields (*i.e.* stationary space-times) play an important role in General Relativity. A stationary gravitational field is one that does not change in time. A stationary space-time exhibits time translation symmetry. This is technically called a time – like killing vector. Many of the gravitational fields which are of high importance in physical theories are, time – independent. The Schwarzschild solutions (exterior and interior) and the Kerr metric of a rotating black hole are common examples of stationary space-times [3-5].

Many researchers are working on stationary space-times and they are trying to know the behavior of the models of the universe and the nature of other physical quantities in it.

Flores *et. al.* [6] have studied the Levi – Civita connection in stationary space-times and applied the results to Kerr space-time. Alias *et. al.* [7] have developed integral formulae for space like hyper surfaces in conformally stationary space-times and discussed their applications. Bartolo *et. al.* [8] have obtained existence and multiplicity results for orthogonal trajectories on stationary space-times under intrinsic assumptions, with some examples and applications. Katz *et. al.* [9] has obtained expression for gravitational energy in stationary space-times. Herberthson [10] has discussed the bounds for and calculation of the multiple moments of stationary space-times. Chrusciel *et. al.* [11] have given solutions of the vacuum Einstein equations with a negative cosmological constant for infinite dimensional families of non-singular stationary space-times. Nayak [12] has investigated the relations between the inertial forces and the Einstein equations in axially symmetric stationary space-times. Shiromizu *et. al.* [13] have shown that strictly stationary space-times cannot have non-trivial configurations of form fields and complex scalar fields. The cosmological models with different physical parameters in stationary space-times have been studied in our earlier work [14-16].

In this research note, we tried to study the pre-Hawking radiation, gravitational collapse and stability of spherically symmetric space-times in time – independent gravitational fields. It is realized that there is no pre - Hawking radiation, there is no stability of the model and it will continue to collapse due to the gravitational effects.

II. THE METRIC AND THE FIELD EQUATIONS

Consider the most general spherically symmetric space-time

$$d\tau^2 = -e^{\lambda}dr^2 - r^2\,d\Omega^2 + e^{\nu}dt^2 \quad \ldots (1)$$

in which ν and λ are functions of *r* and *t* and obeys the relation

$$\lambda + \nu = f(t)$$

where $f(t)$ is a function of time *t* only and $f(t) \leq 0$ all over to space, as it may be found by Landau [17]. Selecting $f(t)$ as $-2\phi(t)$, then we write

$$\nu = -\lambda - 2\phi \quad \ldots (2)$$

The Einstein's field equations

$$R_{ij} - \frac{1}{2} R\, g_{ij} = 8\pi T_{ij} \quad \ldots (3)$$

in stationary space-times takes the form

$$P_{\alpha\beta} + \frac{h}{2} f_{\alpha}^{\ \gamma} f_{\beta\gamma} - \frac{1}{2}\gamma_{\alpha\beta} P = 8\pi T_{\alpha\beta} \quad \ldots 4)$$

$$\frac{3}{8} h\, f_{\alpha\beta} f^{\alpha\beta} + \frac{1}{2}P = \frac{8\pi}{h} T_{44} \quad \ldots (5)$$

$$\frac{\sqrt{h}}{2} f_{\alpha\ \ |\beta}^{\ \beta} + \frac{3}{2} f_{\alpha\beta}\, (\sqrt{\mathrm{h}})^{|\beta} = \frac{8\pi}{\sqrt{h}} T_{4\alpha} \quad \ldots (6)$$

in which

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{4\alpha}\ g_{4\beta}}{h}, \quad \alpha,\ \beta = 1,\ 2,\ 3 \quad \ldots (7)$$

is the three – dimensional metric tensor determining the geometry of space, $f_{\alpha\beta}$ is the three – dimensional antisymmetric tensor given by

$$f_{\alpha\beta} = g_{\beta\,|\,\alpha} - g_{\alpha\,|\,\beta} = \frac{\partial g_{\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha}}{\partial x^{\beta}} \ldots (8)$$

$$h = g_{44}\ , \quad g_{\alpha} = \frac{-g_{4\alpha}}{h}\ , \alpha = 1,\ 2,\ 3 \quad \ldots (9)$$

P is the three - dimensional scalar curvature given by

$$P = \gamma^{\alpha\beta} P_{\alpha\beta} \quad \ldots (10)$$

where $P_{\alpha\beta}$ is the three - dimensional Ricci tensor constructed from the three – dimensional metric tensor $\gamma_{\alpha\beta}$ in the same way as R_{ik} is constructed from the g_{ik} [17].

The differential equations (4 - 6) for the metric (1) are given by

$$8\pi T_1^1 = \frac{1}{r^2}\left\{1 - e^{-\lambda}\right\} \quad \ldots (11)$$

$$8\pi T_2^2 = 8\pi T_3^3 = \frac{1}{2} e^{-\lambda} \frac{\lambda'}{r} \quad \ldots (12)$$

$$8\pi T_4^4 = \frac{1}{r^2} - e^{-\lambda}\left\{\frac{1}{r^2} - \frac{\lambda'}{r}\right\} \quad \ldots (13)$$

$$8\pi T_4^1 = 0 \quad \ldots (14)$$

(Here and hereafter dash ($'$) and ($\cdot$) denotes the derivative with respect to *r* and *t* respectively.)

III. THE STRESS - MOMENTUM TENSOR AND THE FOUR - VELOCITY IN OUR MODEL

The stress - momentum tensor for a perfect fluid is given as

$$T_j^i \;=\; g_{jk}\,(\rho + p)\,u^i u^k \;-\; \eta_j^i\,p \qquad \ldots(15)$$

where ρ and p are the proper energy density and the proper pressure of the perfect fluid, related by the equation of state

$$p = \omega\,\rho\,, \qquad 0 \le \omega \le 1\,. \qquad \ldots(16)$$

The particles (or elements of mass) describe a purely radial trajectory (inwards, or outwards) in our model, so that the only non - vanishing components of four - velocity u^i are u^1 and u^4. Denote $u^4 = \gamma$ and the magnitude of u^i is given by

$$g_{ij}\,u^i u^j \;=\; 1 \qquad \ldots(17)$$

from which, we write

$$1 \;=\; e^{-\lambda - 2\phi}\,\gamma^2 \;-\; e^{\lambda}\,(u_p^1)^2 \qquad \ldots(18)$$

which yields

$$\left|u_p^1\right| \;=\; e^{-\lambda - \phi}\,\gamma\sqrt{1 \,-\, e^{\lambda + 2\phi}\,\gamma^{-2}} \qquad \ldots(19)$$

(where the subindex p stands for "particles"). Here and hereafter we have freely used the notations and symbols of Ribas [18].

For infalling particles $u_p^r \;=\; -\left|u_p^1\right|$ and for outgoing particles $u_p^r \;=\; +\left|u_p^1\right|$.

We call χ the proportion of infalling particles. Hence $1 - \chi$ is the proportion of outgoing particles. The mean "r - velocity" u^1 is given as

$$u^1 = \chi\,\left|u^1\right| + (1 - \chi)\,(-\left|u^1\right|)$$
$$= -(1 - 2\chi)\,e^{-\lambda - \phi}\,\gamma\,\sqrt{1 - e^{\lambda + 2\phi}\,\gamma^{-2}} \qquad \ldots(20)$$

From the equation (15), we write the components of stress - energy tensor T_j^i as

$$T_1^1 \;=\; -(1 - 2\chi)^2 e^{-\lambda - 2\phi}\,\gamma^2\,(1 - e^{\lambda + 2\phi}\,\gamma^{-2})(1 + \omega)\,\rho \;-\; \omega\,\rho \qquad \ldots(21)$$

$$T_2^2 \;=\; T_3^3 \;=\; -\,p \;=\; -\,\omega\,\rho \qquad \ldots(22)$$

$$T_4^4 \;=\; e^{-\lambda - 2\phi}\,\gamma^2\,(1 + \omega)\,\rho \;-\; \omega\,\rho \qquad \ldots(23)$$

$$T_4^1 \;=\; -(1 - 2\chi)\,e^{-2\lambda - 3\phi}\,\gamma^2\,\sqrt{1 - e^{\lambda + 2\phi}\,\gamma^{-2}}\,(1 + \omega)\,\rho \qquad \ldots(24)$$

In the case of ultrarelativistic (the particles moving at the speed very close to speed of light), $\gamma >> 1$, and hence we delete the terms of γ with higher powers in denominator. Thus the above equations (21 - 24) re-write as

$$T_1^1 \;\approx\; -(1 - 2\chi)^2\,e^{-\lambda - 2\phi}\,\gamma^2\,(1 + \omega)\,\rho \qquad \ldots(25)$$

$$T_4^4 \;\approx\; e^{-\lambda - 2\phi}\,\gamma^2\,(1 + \omega)\,\rho \qquad \ldots(26)$$

$$T_4^1 \;\approx\; -(1 - 2\chi)\,e^{-2\lambda - 3\phi}\,\gamma^2\,(1 + \omega)\,\rho \qquad \ldots(27)$$

OR

$$\rho \;\approx\; e^{\lambda + 2\phi}\,\gamma^{-2}\,(1 + \omega)^{-1}\,T_4^4 \qquad \ldots(28)$$

$$T_1^1 \;\approx\; -(1 - 2\chi)^2\,T_4^4 \qquad \ldots(29)$$

$$T_4^1 \;\approx\; -(1 - 2\chi)\,e^{-\lambda - \phi}\,T_4^4 \qquad \ldots(30)$$

(Here and hereafter the symbol $\approx$ stands for approximate value.)

IV. PRE – HAWKING RADIATION, GRAVITATIONAL COLLAPSE AND STABILITY

Vachaspati *et. al*. [2] has demonstrated that collapsing stars emit pre - Hawking radiation, by considering the fact that pre - Hawking radiation spectrum results to be roughly proportional of Hawking radiation of black holes. In view of this, Ribas [18] has deduced a model for ultrarelativistic spherically symmetric pre-Hawking radiating gravitational collapse. For this pre - Hawking radiating gravitational collapse, we have

$$\dot{m} \;\approx\; \frac{-k}{r^2} \qquad \ldots(31)$$

where *k* is a constant of proportionality and *m* denotes the inner mass which is lost as radiation. We are considering not only the global emission of the collapsing star but also that of the inner layers towards the outer ones. We consider the relationship between the function λ and the mass *m*:

$$e^{-\lambda} \;=\; \left(1 \;-\; \frac{2m}{r}\right)$$

which gives

$$\dot{\lambda} \;=\; \frac{2\dot{m}}{r}\,e^{\lambda} \qquad \ldots(32)$$

In stationary space-times, the gravitational potentials g_{ij} are independent of time *t*. Hence, for the metric (1) in stationary space-time, λ is a function of *r* alone. Therefore from equations (31) and (32), we have

$$\dot{\lambda} = -\frac{2ke^{\lambda}}{r^3} = 0 \qquad \ldots(33)$$

yields

$$k = 0 \qquad \ldots(34)$$

If $k = 0$ then equation (31) yield, $\dot{m} = 0$. Hence the mass *m* is constant with respect to time *i.e.*, there is no radiation. This shows that there is no pre - Hawking radiation in case of stationary space-time.

From equation (14) and (30), we have

$$-(1 - 2\chi)e^{-\lambda - \phi} 8\pi T_4^4 = 0$$

OR $$r(1 - 2\chi)e^{-\phi} 8\pi T_4^4 = 0 \qquad \ldots(35)$$

We split the left side of equation (35) into two terms as (in order to make explicit the contribution of infalling and outgoing fluxes):

$$r(1-\chi)e^{-\phi} 8\pi T_4^4 - r\chi e^{-\phi} 8\pi T_4^4 = 0 \qquad \ldots(36)$$

The particle – antiparticle pair is created near the star's horizon due to the vacuum energy. The particle is sucked inside because of the strong gravitational pull of the star and simultaneously the antiparticle created and emitted away causes the pre - Hawking radiation. Since the second term of equation (36) is the term containing infalling matter and equation (33) contains the term of pre - Hawking radiation, therefore we equate this two terms:

$$-r\chi e^{-\phi} 8\pi T_4^4 = -\frac{2ke^{\lambda}}{r^3} \qquad \ldots(37)$$

OR $$\chi = \frac{e^{\lambda+\phi}}{4\pi r^2 T_4^4}\left(\frac{k}{r^2}\right) \qquad \ldots(38)$$

As $k = 0$, $\chi = 0$, which means that the proportion of infalling matter is zero. Hence in this case, all the particle – antiparticle pairs generated near the event horizon annihilate each other, which again strongly suggested that there is no pre - Hawking radiation in case of stationary space-times.

In the stability phase, $e^{-\lambda} << 1$, so that equation (13) infer

$$8\pi T_4^4 \approx \frac{1}{r^2} \qquad \ldots(39)$$

With this value, the equation (35) gives the stability phase (*st*) as

$$r(1 - 2\chi_{st})e^{-\phi_{st}}\left(\frac{1}{r^2}\right) = 0 \qquad \ldots(40)$$

OR $$\chi_{st} = \frac{1}{2} \qquad \ldots(41)$$

Using this stability phase value, equation (29) gives

$$T_1^1 = 0 \qquad \ldots(42)$$

and then from equation (11), we write the stability phase value of e^{λ} as

$$e^{\lambda_{st}} = 1 \quad \text{OR} \quad \lambda_{st} = 0 \qquad \ldots(43)$$

Using the stability phase value of χ_{st} and $e^{\lambda_{st}}$ given by equations (41) and (43) respectively, from equation (38), we arrived at the stability phase value of $e^{-\phi}$ as

$$e^{-\phi_{st}} = 0 \quad \text{OR} \quad \phi_{st} \approx \textit{infinity} \qquad \ldots(44)$$

Our stability value of $e^{\lambda_{st}}$ contradicts the situation of stability that $e^{-\lambda} << 1$. Also ϕ_{st} is infinity shows minus infinity value of ν_{st} (from equation (2)). Thus our model of gravitational collapse does not reach the stability phase in stationary space-time and it is not stable. Also our model has no pre - Hawking radiation and the model will continue to collapse due to the gravitational effect.

V. CONCLUSION

We studied the most general spherically symmetric space-time, in time – independent gravitational fields in related to the pre - Hawking radiation, gravitational collapse and stability of the model. It is seen that in stationary space-time, there is no pre - Hawking radiation, means the model is free from pre - Hawking radiation and the model will continue to collapse due to the gravitational effect. The model is unstable and has no stability phase in stationary space-times.

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